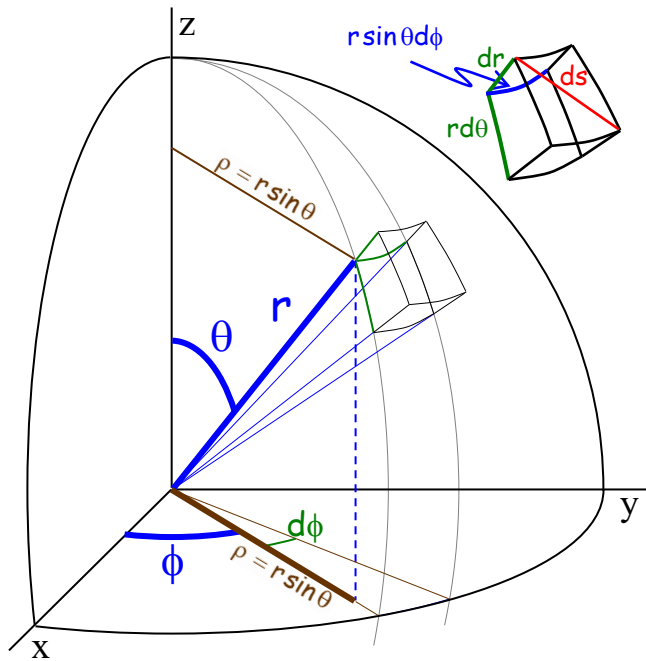


SPHERICAL POLAR COORDINATES

Spherical polar coordinates depend on r , θ and ϕ . The Cartesian coordinates are written as



$$x = \rho \cos \phi = r \sin \theta \cos \phi$$

$$y = \rho \sin \phi = r \sin \theta \sin \phi$$

$$z = r \cos \theta$$

A differential path in 3-D space is

$$ds^2 = dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2$$

The second derivative of a function, ψ is

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} = \frac{1}{r} \frac{\partial^2}{\partial r^2} (\rho \psi) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial \psi}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 \psi}{\partial \phi^2}$$